`scriptRKCQComprehesiveTests.m`

Testing Script for RKCQ methods

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This script serves as a way to test various RKCQ functions. We solve simple interior problems with the wave equation with Dirichlet and Neumann boundary conditions. We begin with a kite shaped domain Ω, its boundary Γ, and a causal signal

$$f(t) = \sin(2t)^9 \chi(t \geq 0).$$

The signal is propagated as a plane wave moving in the direction $\mathbf{d} = (-1/\sqrt{2}, 1/\sqrt{2})$. We assume that the signal is causal (it does not interact with the scatterer at $t = 0$). Therefore, our solution will be causal as well.

Experiment number 1 solves the Neumann problem

$$
\begin{aligned}
\Delta u &= u_{tt} && \text{in} && \Omega \times [0, T] && (1) \\
\partial_\nu u &= \beta_1 && \text{on} && \Gamma \times [0, T] && (2) \\
&&&&&& (3)
\end{aligned}
$$

using an indirect formtulation

$$W * \psi = -\beta_1.$$

We postprocess the solution $u = D * \psi$ with `RKCQforward` on the observation points $(0, -1.25)$ and $(1.5, 0)$ within the interior of the kite. We compute the errors on these observation points at the final time T.

Experiment number 2 solves the following Dirichlet problem

$$
\begin{aligned}
\Delta u &= u_{tt} && \text{in} && \Omega \times [0, T] && (4) \\
\gamma u &= \beta_0 && \text{on} && \Gamma \times [0, T] && (5) \\
&&&&&& (6)
\end{aligned}
$$

with the indirect formulation

$$V * \eta = \beta_0$$

and the postprocessing on the observation points

$$u = S * \eta.$$

We again compute the pointwise errors on the observation points at the final time T.

There are two different modes to run this script. If a variable external HAS NOT been defined, the script requests the user to input: N (spatial discretization parameter), M (time discretization parameter), T (the final time), the time lag, and the experiment number (1 or 2). If the variable external has been defined, these data need to have been declared in advance. In this case, the errors are appended as an additional row to a matrix of errors.