`simulationCylindricalWave.m`

Simulating scattering of a cylindrical wave

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This script is a demonstration of deltaBEM and CQ being used to solve an exterior scattering problem. We will demonstrate scattering by a pair of sound-soft obstacles, which corresponds a Neumann boundary condition.

Problem parameters and formulation. Our scatterers are two disjoint ellipses, the first centered at $(1, 1)$ with x and y semiaxes $1/2$ and 1, and the second centered at $(-1, -1)$ with x and y semiaxes 1 and $1/2$. Let Ω be the disjoint union of these two sets, and let Γ denote their boundary. The geometry is discretized in space with $N = 150$ points per obstacle and in time with $M = 350$ time steps. We decompose the total wave in a non-physical manner into the incident and scattered waves, $u^{tot} = u^{inc} + u^{scat}$. The incident wave is known at all points in space and for all times, and the scattered wave will be the quantity we solve for. For this example, we will solve a Neumann problem. The total wave satisfies the PDE

$$\begin{aligned} \Delta u^{tot} + \delta(\mathbf{x_0}) = u^{tot}_{tt} \qquad & \text{in } \mathbb{R}^d \setminus \Gamma \times [0, T] \\ [\![\partial_\nu u^{tot}]\!] = 0 \qquad\qquad & \text{on } \Gamma \times [0, T]. \end{aligned}$$

If we subtract the known incident wave (which satisfies the wave equation with the point source $\delta(\mathbf{x}_0)$), we arrive at the problem

$$\begin{aligned} \Delta u^{scat} &= u^{scat}_{tt} \qquad \text{in } \mathbb{R}^d \setminus \Gamma \times [0, T] \\ \partial_\nu u^{scat} &= \beta_1 \qquad \text{on } \Gamma \times [0, T] \end{aligned}$$

where we have set $\beta_1 = -\partial_\nu u^{inc}$. We will solve for the unknown scattered field using a double layer indirect ansatz. After computing the scattered field on a set of observation points, we add back the incident field to recover the total field, which is the quantity of interest. The point source $\delta(\mathbf{x}_0)$ in the PDE generates a cylindrical wave emanating from the point x_0. For this simulation, we choose $x_0 = (-1, 0.65)$. We can also choose the signal transmitted by the cylindrical wave, which we take as $f(t) = \sin(16t)^5 H(t)$ where $H(t)$ is the Heaviside function.

Integral represntation and BIE. Our use of a double layer ansatz for the solution leads to the time domain boundary integral equation

$$-\mathcal{W} * \boldsymbol{\eta} = \boldsymbol{\beta}_1$$

where we solve for the unknown (and non-physical) density $\boldsymbol{\psi}$ and then post-process to compute the scattered wave

$$u^{scat} = \mathcal{D} * \boldsymbol{\psi}.$$

This is done by using PDEtool to generate a mesh in the domain $[-2, 2]^2 \setminus \overline{\Omega}$. To avoid storing a single enormous matrix (corresponding to the discrete layer potential $\mathcal{D}$) in memory, we partition the domain into blocks of 200 points and evaluate u^{scat} at all times on these 200 observation points at a time. We only observe in the box $[-2, 2]^2$, but this is easily modified by the user. After computing the scattered wave with the potential representation on all of the prescribed points, we compute the incident wave at all of the observation points and add the scattered and incident wave together, which we then plot at each discrete time.