`simulationPlanewave.m`

Simulating scattering of a planewave

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This script is a demonstration of deltaBEM and CQ being used to solve an exterior scattering problem. When run, it will produce a movie of scattering by a sound-hard obstacle (corresponding to a Dirichlet boundary condition).

Problem parameters and formulation. Our scatterer Ω will be a non-convex kite shape. We let Γ denote its boundary. This is discretized in space with $N = 250$ points and in time with $M = 300$ time steps. Data for the problem will be a plane wave traveling in the direction $\mathbf{d} = (1, 0)$ and transmitting the signal

$$f(t) = \sin(16t)\chi(t \geq 0)$$

where $\chi(t)$ is a smooth cutoff function in the time variable. The incident wave is lagged by one second in time to prevent the simulation from starting with the incident wave already interacting with the scatterer. The total wave is decomposed in a non-physical manner into the incident and scattered waves, $u^{tot} = u^{inc} + u^{scat}$. The incident wave is known at all points in space and for all times, and the scattered wave will be the unknown quantity we compute. The total wave satisfies the PDE

$$\begin{aligned} \Delta u^{tot} = u^{tot}_{tt} & \qquad \text{in } \mathbb{R}^d \setminus \Gamma \times [0, T] \\ [\![\gamma u^{tot}]\!] = 0 & \qquad \text{on } \Gamma \times [0, T]. \end{aligned}$$

If we subtract the known incident wave (which satisfies the wave equation in free space), we arrive at the problem

$$\begin{aligned} \Delta u^{scat} &= u^{scat}_{tt} \qquad \text{in } \mathbb{R}^d \setminus \Gamma \times [0, T] \\ \gamma u^{scat} &= \beta_0 \qquad \text{on } \Gamma \times [0, T] \end{aligned}$$

where we have set $\beta_0 = -\gamma u^{inc}$. We will solve for the unknown scattered field using a single layer indirect ansatz. After computing the scattered field on a set of observation points, we will add back the incident field so that we may plot the total field.

Integral representation and BIE. Our use of a single layer ansatz for the solution leads to the time domain boundary integral equation

$$\mathcal{V} * \boldsymbol{\eta} = \boldsymbol{\beta}_0$$

where we solve for the unknown (and non-physical) density $\boldsymbol{\eta}$ and then post-process to compute the solution

$$u^{scat} = \mathcal{S} * \boldsymbol{\eta}.$$

This is done by using PDEtool to generate a mesh in the domain $[-2,2]^2 \setminus \overline{\Omega}$. To avoid storing a single enormous matrix (corresponding to the discrete layer potential $\mathcal{S}$) in memory, we partition the domain into blocks of 200 points and evaluate u^{scat} at all times on these blocks 200 observation points. We only observe in the box $[-2,2]^2$, but this is easily modified by the user. After computing the scattered wave with the potential representation, we compute the incident wave at all of the observation points and add the scattered and incident wave together, which we then plot at each discrete time.