# MATH 353: Engineering Mathematics III – Section 012

Spring 2014 (F.–J. Sayas) Homework #2 Due February 28

**Instructions.** The coding part of the exercises has to be turned in as Matlab output (copy-paste from the Matlab window and/or editor). You can try and figure out how to use the Matlab Publish utility. (Google it!). Bring the homework to Friday's lecture. (That's collecting point and time. If you cannot make it, give it to someone else. If you cannot make that either, *tell me in advance* (no late homework!) and we'll make it work.)

1. (By hand – 4 points) Review: working with arbitrary magnitude (any exponent) but four-digit precision floating point, compute

$$\sqrt{x^3+2}-\sqrt{x^3+3} \qquad \text{and} \qquad \frac{-1}{\sqrt{x^3+2}+\sqrt{x^3+3}}$$

   for $x = 100$. The rules are: each number has to be rounded-off to four digits, after each simple computation. It is not fine to compute everything and then round-off the final result.

2. (By hand – 4 points) How many iterations of the bisection method do you need if you start in the interval $[3, 6]$ and you want $|r - r_{\text{app}}| \le 10^{-6}$? Here $r$ is the root that the bisection method is going to find and $r_{\text{app}}$ is the approximation given by the method after $n$ iterations.

3. (By hand – 4 points) **A mathematically tricky question.** A continuously differentiable function $g$ has exactly three fixed points $r = 1$, $r = 2$ and $r = 3$. We know that $g'(1) = g'(3) = \frac{1}{2}$. Show that $g'(2) > 1$. The argument can be made with a correctly drawn graph, by understanding what fixed points are and what a derivative means.

4. (By hand – 4 points) Write the mathematical expression for the following function

```
f = @(x) x.^2.*(x>0).*(x<1)+(1+x).^2.*(x>=1);
```

5. (By hand – 4 points) Write an algorithmic description (that is, write the iteration with mathematically meaningful formulas) of the following iterative process:

```
x=1;
for i=1:2e+2
    x=0.5*(x+3/x);
end
```

   (I'm asking also how many iterations you get to do when you run this piece of code.)

6. (Computer – 4 points) Write an anonymous Matlab function corresponding to

$$f(x) = \frac{\cos(10\,x^2)}{1+x^2}$$

and plot it in $[-4, 4]$, using `plot` and `fplot`. Give the instructions you had to use to produce the plots. I care more about the commands than about the plot itself.

7. (Computer – 4 points) We now create the list

```
list = 1:0.1:2;
```

How many element does the array `list` have? Find out what the following quantities are

```
list(3)          list(3:5)          list(1:2:end)
```

You should be able to figure it out before you type the question to get the result.

8. (Computer – 4 points) **Just run the code.** Use the code `bisection.m` to find the root to six correct decimal places of the following equations

$$x^3 = 9 \qquad \text{and} \qquad \cos^2 x + 6 = x.$$

(You will first need to figure out an interval where you can guarantee the algorithm to find a solution.)

9. (By hand & computer – 8 points) You are going now to build an m–file `babylonian.m` containing *specifically designed code* for the following iteration

$$x_0 = a, \qquad x_{i+1} = \frac{1}{2}\Big(x_i + \frac{a}{x_i}\Big) \qquad i = 1, \ldots, \texttt{itmax}$$

Here $a > 0$ is a parameter. (By specifically designed, I mean that the code has to solve this problem and nothing else. Do not use the general code of fixed point iteration for this.)

- Show (with math) that if this iteration converges, it converges to $\sqrt{a}$.
- Show that $x_i > 0$ for all $i$.
- Prepare a function with following prototype

```
function [sqa,err]=babylonian(a,itmax)
```

- The output is `sqa`, the approximation of $\sqrt{a}$ after `itmax` iterations, and an array `err` with the quantities $|x_i - \sqrt{a}|$.
- Run it for $a = 3$ and check your result.

Read about this method in your book.(Example 1.6.)