MATH 353: Engineering Mathematics III – Section 012

Spring 2014 (F.–J. Sayas) Homework #5 Due March 28

1. (Review - do by hand – 5 points) Assume that we are working with floating point numbers with only 5 digits. Every single operation you perform has to be rounded-off to five significant digits (the exponents are not an issue). Show what you get when you compute
$$\frac{f(1.0001)-f(1)}{0.0001} \qquad \frac{f(1.00001)-f(1)}{0.00001}$$
if $f(x)=x^3$.

2. (Review – do by hand – 5 points) Write the Newton formula for interpolation polynomial at the points
$$(1,1), \qquad (2,2), \qquad (3,2), \qquad (4,1).$$
Evaluate it at 3/2 and evaluate its derivative at the same point.

3. (By hand – 5 points) Exercise 5.1 in the book (page 252)

4. (By hand – 5 points) Exercise 5.2 in the book (page 252)

5. (Computer – 10 points) We want to check this double forward difference formula to approximate the derivative.
$$\frac{-f(x_0+2h)+4f(x_0+h)-3f(x_0)}{2h} \approx f'(x_0)$$
   (a) Write a testing device (like the functions `testFwdDiff` and `testCentDiff` in the Lab) to test that
   $$E_h = \left|\frac{-f(x_0+2h)+4f(x_0+h)-3f(x_0)}{2h} - f'(x_0)\right| \approx C\,h^2$$
   (b) Check it with the function $f(x)=\exp(x)$ at $x_0=1$. Do experiments with $h=0.1, 0.01, 0.001$ and verify that you get what you expect.
   (c) Do a loglog plot of (h, E_h) where $h=[0.5, \quad 0.25, \quad 0.125, \quad ..., \quad 2^{-10}]$. In the loglog plot, use circular markers and lines. On top of this plot, show the line (h,h^2) to verify that the predicted behavior.

6. (By hand – 5 points) Recall the Taylor expansion formula
$$f(x+h)=f(x)+hf'(x)+h^2\tfrac{1}{2}f''(x)+h^3\tfrac{1}{6}f'''(x)+h^4\tfrac{1}{24}f^{iv}(c),$$
where c is unknown between x and $x+h$. Use it to show that
$$\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}=f''(x_0)+\mathcal{O}(h^2),$$
where by $\mathcal{O}(h^2)$ we mean something that can be bounded above by Ch^2.

7. (Computer – 5 points) Follow the steps of Problem 5 to show that
$$E_h = \left|\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2} - f''(x_0)\right| \approx C\,h^2$$
for the function $f(x)=\log(x)$ at $x_0=1$ (Recall that for us –and for Matlab– log is the natural logarithm.)