MATH 353: Engineering Mathematics III – Section 012

Spring 2014 (F.–J. Sayas) | Homework #6 | Due April 11

1. (Review – by hand – 3 points) Write the Lagrange formula for the interpolation polynomial at the points
$$(0,1), \qquad (1,2), \qquad (2,\tfrac{1}{7}), \qquad (3,-\tfrac{2}{3}).$$

2. (Review – by hand – 3 points) Compute two steps of Newton's method to try to approximate a root of $f(x) = x^4 - 3$ starting at $x_0 = 1$.

3. (Review – by hand – 4 points) Show that
$$\frac{u(x_0) - 2u(x_0 - h) + u(x_0 - 2h)}{h^2} = u''(x_0) + \mathcal{O}(h).$$

4. (By hand – 5 points) Find the degree of precision of the formula
$$\int_{-1}^{1} f(x)\mathrm{d}x \approx f(-1/\sqrt{3}) + f(1/\sqrt{3}).$$
(**Hint.** Compare the exact and approximate values for $f(x) = 1$, $f(x) = x$, $f(x) = x^2$, ... until they are different.)

5. (By hand – 5 points) Compute α, β, γ so that the following approximation
$$\frac{\alpha f(x_0 - h) + \beta f(x_0) + \gamma f(x_0 + 2h)}{h} \approx f'(x_0)$$
is of order two. (**Hint.** Subtitute with the Taylor expansions
$$\begin{aligned} f(x_0 + 2h) &= f(x_0) + 2hf'(x_0) + \frac{(2h)^2}{2} f''(x_0) + \frac{(2h)^3}{6} f'''(c_1), \\ f(x_0 - h) &= f(x_0) - hf'(x_0) + \frac{h^2}{2} f''(x_0) - \frac{h^3}{6} f'''(c_2) \end{aligned}$$
and try to find conditions satisfied by α, β and γ.)

6. (Computer – 5 points) In the last Lab you should have finished with three functions (the first one was provided)

```
midpointrule(f,interval,m)
traepozidrule(f,interval,m)
simpsonrule(f,interval,m)
```

Give the code for the last two ones. *The code has to contain the help lines!*

7. (Computer – 5 points) In the same graph you are going to show the errors of the three formulas when you try to compute the integral

$$\int_0^1 xe^x \mathrm{d}x,$$

using $m2, 4, 8, 16, 32, ..., 128$ subdivisions of the interval. Plot the errors in the same loglog plot using circular markers and lines: the error for the midpoint rule in blue, trapezoid in red, Simpson in black.

8. (Computer – 5 points) With your code `simpsonrule` and $m = 1$, check that the Simpson rule integrates exactly

$$\int_0^3 (x^3 - 4x + 1)\mathrm{d}x.$$

9. (Computer – 5 points) If $M_h(f)$ is the composite midpoint rule, $T_h(f)$ is the composite trapezoid rule and $S_h(f)$ is the composite Simpson rule, choose a function (not a polynomial) and m and check that

$$S_h(f) = \frac{2}{3}M_h(f) + \frac{1}{3}T_h(f),$$

that is, compare the result of the computations in both sides of the equality above.

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Spring 2014 (F.–J. Sayas) Homework #5 Due March 28

1. (Review - do by hand – 5 points) Assume that we are working with floating point numbers with only 5 digits. Every single operation you perform has to be rounded-off to five significant digits (the exponents are not an issue). Show what you get when you compute
$$\frac{f(1.0001)-f(1)}{0.0001} \qquad \frac{f(1.00001)-f(1)}{0.00001}$$
if $f(x)=x^3$.

2. (Review – do by hand – 5 points) Write the Newton formula for interpolation polynomial at the points
$$(1,1), \qquad (2,2), \qquad (3,2), \qquad (4,1).$$
Evaluate it at 3/2 and evaluate its derivative at the same point.

3. (By hand – 5 points) Exercise 5.1 in the book (page 252)

4. (By hand – 5 points) Exercise 5.2 in the book (page 252)

5. (Computer – 10 points) We want to check this double forward difference formula to approximate the derivative.
$$\frac{-f(x_0+2h)+4f(x_0+h)-3f(x_0)}{2h} \approx f'(x_0)$$
   (a) Write a testing device (like the functions `testFwdDiff` and `testCentDiff` in the Lab) to test that
   $$E_h = \left|\frac{-f(x_0+2h)+4f(x_0+h)-3f(x_0)}{2h} - f'(x_0)\right| \approx C\,h^2$$
   (b) Check it with the function $f(x)=\exp(x)$ at $x_0=1$. Do experiments with $h = 0.1, 0.01, 0.001$ and verify that you get what you expect.
   (c) Do a loglog plot of (h, E_h) where $h = [0.5, \quad 0.25, \quad 0.125, \quad ..., \quad 2^{-10}]$. In the loglog plot, use circular markers and lines. On top of this plot, show the line (h, h^2) to verify that the predicted behavior.

6. (By hand – 5 points) Recall the Taylor expansion formula
$$f(x+h) = f(x) + hf'(x) + h^2\tfrac{1}{2}f''(x) + h^3\tfrac{1}{6}f'''(x) + h^4\tfrac{1}{24}f^{iv}(c),$$
where c is unknown between x and $x+h$. Use it to show that
$$\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2} = f''(x_0) + \mathcal{O}(h^2),$$
where by $\mathcal{O}(h^2)$ we mean something that can be bounded above by Ch^2.

7. (Computer – 5 points) Follow the steps of Problem 5 to show that
$$E_h = \left|\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2} - f''(x_0)\right| \approx C\,h^2$$
for the function $f(x)=\log(x)$ at $x_0=1$ (Recall that for us –and for Matlab– log is the natural logarithm.)