MATH 353: Engineering Mathematics III – Section 012

Spring 2014 (F.–J. Sayas) | Homework #9 | Due May 9

Important. Whenever you write a function, don't forget to include the help lines. The axes in all plots have to be labeled.

1. (Computer – 5 points) Plot in the same figure the graphs of

$$\sin x, \qquad \cos x \qquad \sin^2 x, \qquad \cos^2 x \qquad \text{for} \qquad 0 \le x \le 2\pi.$$

Use `legend` to show which is which in the plot .

2. (Computer + By hand – 5 points) Use Matlab to solve the linear system

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 2 \end{bmatrix}.$$

Solve it by hand, using forward substitution, and check that you got the right result.

3. (Computer + By hand – 5 points) Use Matlab to solve the linear system

$$\begin{bmatrix} -1 & 2 & 1 & 1 \\ 0 & -1 & 3 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5/2 \\ -4 \\ -1 \\ 6 \end{bmatrix}.$$

Solve it by hand, using back substitution, and check that you got the right result.

4. (By hand + Computer – 5 points) Consider the following matrix

$$\mathrm{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Find out what you obtain when you multiply P times a general column vector

$$\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)^\top = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}.$$

(The symbol $\top$ is used to transpose, meaning that what your vector $\mathbf{x}$ is a column vector. It corresponds in Matlab to `'`) Use Matlab to check that your result is correct by multiplying $\mathrm{P}\mathbf{x}$ where $\mathbf{x} = (1, 2, 3, 4, 5)^\top$.

5. (Computer – 5 points) Write the Matlab lines to generate a **sparse** $N \times N$ matrix with the following shape

$$\begin{bmatrix} 4 & -1 & 2 & & & & \\ 3 & 4 & -1 & 2 & & & \\ & 3 & 4 & -1 & 2 & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & & & 3 & 4 & -1 & \\ & & & & 3 & 4 & \end{bmatrix}$$

6. (Computer – 15 points) Use the code `FiniteDiffBVP` that you should have produced in Lab # 11, to solve the boundary value problem

$$-y'' + \frac{1}{1+x^2}\, y = 1 + 11x^2, \qquad 0 \le x \le 1, \qquad y(0) = 1, \qquad y(1) = 0.$$

The exact solution is $y(x) = 1 - x^4$.

(a) Include your code for `FiniteDiffBVP`.

(b) Compare the graphs of the exact and approximate solutions with $N = 100$ interior points.

(c) Compute the approximate solution with $N = 10, 20, 40, 80, 160$ points and check that the errors

$$E_h = \max_{0 \le i \le N+1} |y_i - y(x_i)|$$

behave like $E_h = \mathcal{O}(h^2)$.