MATH 353: Engineering Mathematics III – Section 012

Spring 2014 (F.–J. Sayas) Lab # 13 May 16

1. We are given a square matrix

$$\mathrm{A} = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 1 & 1 & 3 \\ 3 & 1 & 1 & 0 \end{bmatrix}.$$

Use the Matlab inbluilt function `lu` and produce a lower triangular matrix L, an upper triangular matrix U and a permutation matrix P, such that PA = LU. Once you have these three matrices, verify that PA = LU.

2. Do the same for an $N \times N$ matrix

$$\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix},$$

built and stored as a *sparse matrix*. (Use `spdiags` for this.) Copy the results for $N = 5$. If you want to see the numbers as fractions, try `format rat`.

3. Take now a general square matrix A. You are going to build a Matlab function

```
function y=powermethod(A,x,M)
```

that computes the following iteration: a vector $\mathbf{x}_0$ is given and we then compute

$$\mathbf{z}_{n+1} = \mathrm{A}\mathbf{x}_n, \qquad \mathbf{x}_{n+1} = \frac{1}{\max z_{n+1,i}}\mathbf{z}_{n+1}, \qquad n = 1, \ldots, M.$$

It is possible to show that, with probability one, the vectors $\mathbf{x}_n$ converge to a vector $\mathbf{x}$ such that $\mathrm{A}\mathbf{x} = \lambda\mathbf{x}$, where λ is the eigenvalue of A with largest absolute value. To do:

   (a) Build the function `powermethod`

   (b) Use the matrix of Exercise 2 with $N = 5$ and check that with increasing values of M and, independently of what $\mathbf{x}_0$ is, you get to the same vector $\mathbf{y}$. (Call me when you are done with this.)

   (c) Check then that $\mathrm{A}\mathbf{y}$ and $\mathbf{y}$ are proportional to each other. The proportion is the eigenvalue.

   (d) Repeat the exercise and compute the largest eigenvalue of the matrix of Exercise 2, for increasing $N = 5, 6, 7, ..., 20$. Let's call it λ_N. Plot (N, λ_N).