MATH 353: Engineering Mathematics III – Section 012

Spring 2014 (F.–J. Sayas) Lab # 6 March 21

1. Check numerically the accuracy of the following approximation of the second derivative

$$f''(x_0) \approx \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}.$$

   To do that compare the results for the function $f(x) = \exp(2x)$ at $x_0 = 1$. (You just need to make some small modifications to today's class script.)

2. Check numerically the accuracy of the following double backward differentiation formula:

$$f'(x_0) \approx \frac{\frac{3}{2}f(x_0) - 2f(x_0-h) + \frac{1}{2}f(x_0-2h)}{h}.$$

   To do that compare the results for the function $f(x) = \exp(2x)$ at $x_0 = 1$. (You just need to make some small modifications to today's class script.)

3. Following what we have done today. You have a sequence of errors E_h depending on a parameter h. They are shown in the following table (h on the left column, E_h on the right):

```
>> [h' err']
ans =
   0.500000000000000   0.250000000000000
   0.250000000000000   0.062500000000000
   0.125000000000000   0.015625000000000
   0.062500000000000   0.003906250000000
   0.031250000000000   0.000976562500000
   0.015625000000000   0.000244140625000
   0.007812500000000   0.000061035156250
   0.003906250000000   0.000015258789063
   0.001953125000000   0.000003814697266
   0.000976562500000   0.000000953674316
```

   We claim that $E_h \approx Ch^p$ for some p to be determined.

   (a) Justify the formula

$$\log\left(\frac{E_{h_1}}{E_{h_2}}\right) \approx p \log\left(\frac{h_1}{h_2}\right)$$

   (b) Use it to figure out what p is.

   (c) Make a loglog plot of (h, E_h). (You need to get a check from me here.) What is the slope of the line you obtain?