MATH 353: Engineering Mathematics III – Section 012

Spring 2014 (F.–J. Sayas) Lab # 8 April 11

Open Matlab and *move to the Desktop or to a folder where you can find your work* at the end of the session. Type these lines

```
>> diary myworkApril11
>> format long
>> format compact
```

Download the function `heun.m` and the script `scriptApril11.m` from my website.

1. The function `heun.m` corresponds to the implementation of Heun's method (the explicit trapezoidal method) to numerically solve

$$y' = f(t,y) \quad a \le t \le b, \qquad\qquad y(a) = y_a.$$

The script `scriptApril11.m` shows you one example of how this works. We are trying to solve the differential equation

$$(t^2+1)y' + 3ty = 6t, \qquad y(0) = 1.$$

First of all, we need to write it in explicit form

$$y' = \frac{6t - 3ty}{t^2+1}, \qquad y(0) = 1.$$

The exact solution of this problem is

$$y(t) = 2 - \frac{1}{(t^2+1)^{3/2}}.$$

- Run the script and figure out what we did. What is the interval where we solved the equation? How many time steps? Some place in the script, we have computed

$$E_h = \max_{0\le j\le n} |w_j - y(t_j)|.$$

What is this value?

- Run the code again with $n = 20$ time steps in the same time interval.
- Run the code again with $n = 100$ time steps in the interval $[0, 10]$.

2. Let us now compute errors for increasing values of n. Run the same example in the interval $[0, 10]$, with $n = 10, 20, 40, 80, 160, 320$, compute the errors, make a loglog plot of the errors and compare them with a loglog plot of (h, h^2). To help you get organized...

- Create a list `listn=[10 20 ...]`
- Compute the vector with all values of h (you'll need it for the plots)

- Run `heun` when `n` takes values in the list `listn`
- Compute the error and accumulate it on a vector of errors.
- Do the loglog plots.

3. As we saw in class, there's an easy modification of this code that gives you the code for Euler's method. In this case the error is $E_h = \mathcal{O}(h)$, as opposed to $E_h = \mathcal{O}(h^2)$ in Heun's method.

   - Create the function `euler.m` following the same model as `heun.m` (copy-paste at will). (Be careful: there's a function `euler.m` in the website that does something different. Don't use it.)
   - Repeat the experiment of Exercise 2 and show that you have order one and not two.

4. We are now going to experiment with a more complicated equation. For this one, we do not know the solution:

$$y' = y \cos t, \qquad 0 \leq t \leq 8\pi \qquad y(0) = 1.$$

   Solve this for $n = 25, 50, 100, 200, 400$ using Heun's method. At the point of getting the result I'm asking you to do the following:

   - Compute the solution for $n = 25$ and plot it. Pause (the command `pause` will wait for you to click on enter) and hold on.
   - Compute the solution for $n = 50$ and plot it.
   - Go on until you have all experiments on the same graph.

5. Repeat Exercise 4 with Euler's method.