
MATH 353: Engineering Mathematics III – Section 012

Spring 2014 (F.-J. Sayas)

Lab # 9

April 18

Open Matlab and *move to the Desktop or to a folder where you can find your work* at the end of the session. Type these lines

```
>> diary myworkApril18
>> format long
>> format compact
```

Download the function `rk4.m` and the script `scriptApril18.m` from my website.

1. First of all, let us check that the RK4 method is actually of order four. Take the equation

$$\frac{y'}{2y^2} + t + 1 = 0, \quad 0 \leq t \leq 2, \quad y(0) = 1,$$

whose exact solution is $y(t) = \frac{1}{(t+1)^2}$. Using $n = 10, 20, 40, 80, \dots, 256$ points, compute, tabulate and plot the errors

$$E_h = \max_{0 \leq i \leq n} |w_i - y(t_i)|$$

and check that $E_h = \mathcal{O}(h^4)$. **Warning.** The function `rk4` returns a column vector in \mathbf{w} , so be careful when you compare with $\mathbf{y}(\mathbf{t})$, \mathbf{t} being a row vector. The solution is very simple: \mathbf{t}' is a column vector with the same values as \mathbf{t} .

2. Using the example in the script `scriptApril18`, carry out the necessary modifications to code the damped forced pendulum equation

$$\theta'' + \frac{g}{L} \sin \theta + d\theta' = m(t) \quad 0 \leq t \leq T_{\max}, \quad \theta(0) = \theta_0, \quad \theta'(0) = 0.$$

The way you have to do this is:

- Set up the physical parameters: g , L , d , the final time T_{\max} , and an amplitude parameter A . (See values below.)
- Define the forcing term $m(t)$ as a function of the variable t . Take

$$m(t) = A \sin t.$$

- Define then the function $f(t, y)$ for the associated first order system.
- Run RK4 and plot the results as shown in the script. Do you understand all the plotting commands? For this first experiment take:

$$g = 9.81, \quad L = 0.3, \quad d = 0, \quad A = 0, \quad T_{\max} = 5.$$

You should see periodic motion.

- Repeat again with $d = 1$. Can you see the effect of the damping parameter?

- Repeat again with $d = 0$ and $A = 5$.

3. **A rotating planet.** Given a massive sun and a much smaller object subject to its gravitational force, the equations of motion for the smaller object are (with dimensionless variables):

$$x'' = \frac{-x}{(x^2 + y^2)^{3/2}}, \quad y'' = \frac{-y}{(x^2 + y^2)^{3/2}}.$$

The sun is located at $(0, 0)$ and assumed not to move. The motion is determined by initial conditions

$$x(0) = x_0, \quad y(0) = y_0, \quad x'(0) = v_0^x, \quad y'(0) = v_0^y.$$

- Write the previous system as a system of four first order differential equations in the variable

$$\mathbf{z} = (x, y, x', y').$$

- Write a script where you run the simulation of the motion of the planet and plot it using `comet`. Run several simulations to see if by changing the initial conditions you can find different kinds of orbits.