MATH 353: Engineering Mathematics III – Section 012

Spring 2013 (F.–J. Sayas) | Homework #3 | Due March 1

Instructions.

- The coding part of the exercises has to be turned in as Matlab output (copy-paste from the Matlab window and/or editor). You can try and figure out how to use the Matlab Publish utility. (Google it!).
- Please use `format compact` to avoid unnecessary blank lines. Give numbers with all possible digits using `format long`. Finally, use the semicolon `;` to hide computations that are not needed.
- Bring the homework to Friday's lecture. (That's collecting point and time. If you cannot make it, give it to someone else. If you cannot make that either, *tell me in advance* (no late homework!) and we'll make it work.)

1. **Easy warm-up.** The following questions are Matlab-related:

   (a) In Matlab, define the anonymous function

   $$u(x) = \frac{x^2+1}{\sin^2(4\,x)+1}$$

   Call it u. Use then the command `fplot` to get the graph of u in the interval $[-2,2]$. (Be careful with brackets in your expressions.)

   (b) What is the result of running the following lines of code and why?

   ```
   >> list=1:0.2:3;
   >> list([1 5 7])
   ```

   (c) I just run this piece of code

   ```
   >> f = @(x) x*(1+x);
   >> f([0 1 2])
   ??? Error using ==> mtimes
   Inner matrix dimensions must agree.

   Error in ==> @(x)x*(1+x)
   ```

   There's is an error message, because there's something wrong in my code. What exactly is it? How would you fix it?

2. **Understading convergence.** Two different methods, trying to compute a particular number we know, have produced the following lists of errors.

```
err1 =
   0.020746217944410
   0.006076689632185
   0.001779827797216
   0.000521299658575
   0.000152685139140
```

and

```
err =
   1.354248688935409
   0.229248688935409
   0.009139993283235
   0.000015733125699
   0.000000000046779
```

(a) Show that the first list corresponds to a method with linear convergence. What is the rate of convergence?

(b) Show that the second list of errors corresponds to a method with quadratic convergence.

3. **Work by hand. No computer.** Exercise 1.4.1 (a) and (b) of the book. (Page 58).

4. **Run the code.** Using the `newton.m` function (downloadable from my website), solve Computer exercise 1.4.2 in the book. (Page 59)

5. **Run the code and discuss.** Run the `newton` function to find the quite obvious roots of the functions

$$p(x) = (x-3)^2, \qquad q(x) = (x-5)^3.$$

Show that convergence is linear in both cases. What is the rate of convergence in each case?

If you haven't done it yet, please read Section 1.4.2 of the book.

6. **A longer problem to finish.** Consider the function

$$p(x) = e^x\,(x-3)^2.$$

(a) Working out the algebra (no computer), show that one Newton iteration of this method is exactly as computing

$$x_{i+1} = x_i - \frac{x_i - 3}{x_i - 1},$$

and therefore

$$e_{i+1} = M_i e_i \qquad e_i = |x_i - 3|, \quad \text{where} \quad M_i \to \tfrac{1}{2} \text{ if } x_i \to 3.$$

This clearly shows that convergence for Newton's method is linear for this case, which is similar to what you should have already observed in Problem 5.

(b) Surprisingly enough, for double roots ($f(r) = f'(r) = 0$ but $f''(r) \neq 0$), the following modification of Newton's method works:

$$x_{i+1} = x_1 - 2\,\frac{f(x_i)}{f'(x_i)}$$

and gives quadratic convergence. You do not have to recode Newton's method for this. Instead of f and f', you just have to send the functions f and $f'/2$ to the `newton` iteration. *Do it in Matlab and check that convergence is quadratic.*