MATH 353: Engineering Mathematics III – Section 012

Spring 2013 (F.–J. Sayas) Homework #3 bis Not collected/not graded

These are exercises so that you can practice for your exam. Before you start, this would be a great time to read Sections 3.1.1 and 3.1.2 in the book.

1. **A very logical question.** You are given three points

$$(x_1, y_1), \quad (x_2, y_2), \quad (x_3, y_3)$$

where the x_i are pairwise different. How many different polynomials of degree three or less go through these three points? (**Hint.** You are free to choose a foruth point and interpolate.)

2. **Understanding Newton's formula.** Imagine that we have points

$$(x_1, f(x_1)), \quad (x_2, f(x_2)), \quad (x_3, f(x_3)), \quad (x_4, f(x_4))$$

and that

$$f[x_1, x_2, x_3] = 0, \qquad f[x_1, x_2, x_3, x_4] = 0.$$

What is the actual degree of the interpolation polynomial through these points? Can you interpret this result graphically?

3. **Comparing methods.** (Exercises 3.1.1 and 3.1.2) For the following collections of points:

   (a) $(0, 1), (2.3), (3, 0)$
   (b) $(-1, 0), (2, 1), (3, 1), (5, 2)$
   (c) $(0, -2), (2, 1), (4, 4)$

   do the following:

   - Write down the interpolating polynomial using Lagrange's formula. Do not simplify the result!
   - Evaluate the previous formula at the point $x = 1$.
   - Compute the divided differences and write the interpolating polynomial in Newton's form. Do not simplify the result!
   - Evaluate the last polynomial you got in $x = 1$.

   If you got everything right, the values at $x = 1$ of both formulas (for each problem) should be the same. Why?

4. Working by hand, do the nested evaluation of the polynomial

$$2 - 3(x - 1) + 4(x - 1)(x + 5) + 3(x - 1)(x + 5)(x - 4),$$

at the points $x = 2$ and $x = 3$.

5. What does the following piece of code do? In particular, can you write down exactly what polynomial are we plotting, where and why?

```
>> xx=0:0.01:1;
>> yy=evaluatelagrange([0 0.5 1],[0 0 1],xx);
>> plot(xx,yy)
```

6. If `n` is a positive integer, what numbers are we computing with these lines?

```
>> a=-1;b=1;
>> x=a+(0:1:n)*(b-a)/n;
```

   Write a mathematical formula for general `a` and `b` and then substitute.

7. How about these ones?

```
>> a=-1;b=1;
>> x=(a+b)/2 + (b-a)/2*cos( (2*(1:n+1)-1)*pi/(2*n+2) );
```

   Write a mathematical formula for general `a` and `b` and then substitute.