MATH 353: Engineering Mathematics III – Section 012

Spring 2013 (F.–J. Sayas) Homework #4 Due March 15

1. (Computer) Define a function
$$f(x) = x \sin\left(\frac{1}{x}\right)$$
and evaluate it *simultaneously* at the points $0.01, 0.02, 0.03, \ldots, 0.5$.

2. (Computer + explanation) Explain what the following commands do:

```
>> m1=linspace(0,2,10);
>> m2=linspace(0,2,11);
```

   In addition to running the lines, explain how many elements you get and what these elements are.

3. (Computer + explanation) Explain what the following lines do:

```
>> A=[1 2 3 4;5 6 7 8;-1 -2 -3 -4];
>> A(:,2)+A(:,3)
>> A(3, end:-1:1)
>> size(A)
```

   Once again, run the lines and explain what happens.

4. (Computer) Consider the following interpolation points
$$(0,-1), \quad (1,2), \quad (\tfrac{3}{2},-1), \quad (2,2), \quad (\tfrac{1}{2},-1), \quad (3,2).$$
   (a) Using the function `evaluatelagrange` and a collection of points in the interval $[0,3]$, make a plot of the interpolation polynomial in these points. On top of the graph, plot the interpolation points. (Recall that we can put a circular marker on points by doing `plot(x,y,'o')`.
   (b) Repeat the exercise using now the divided difference function `divideddiff` and the nested evaluation of polynomials `nested`.

5. (By hand) We are given five points, in this particular order:
$$(1,-1), \qquad (2,0), \qquad (3,3), \qquad (4,8), \qquad (0,0).$$
   (a) Compute the divided differences corresponding to these points. Once you are done, compare with what you get using `divideddiff`.
   (b) Write the quartic interpolation polynomial at these points. What is the effective degree of this polynomial? Can you say anything about where these five points are placed?

6. (Computer) Consider the points

$$(0,1), \quad (\tfrac{1}{2},0), \quad (1,-1), \quad (\tfrac{3}{2},-2), \quad (2,-1), \quad (\tfrac{5}{2},0), \quad (3,1).$$

   (a) Plot the points with circular markers.

   (b) On top of these points, plot the interpolation spline in the interval $[0,3]$. Use the format `yy=spline(x,y,xx)` to evaluate the spline at many points.

7. (By hand) Here are some lines we just run with a script:

```
>> x=0:5;
>> f = @(x) x.^3;
>> y = f(x);
>> S=spline(x,y);
>> A=S.coefs
A =
    1.0000   -0.0000    0.0000         0
    1.0000    3.0000    3.0000    1.0000
    1.0000    6.0000   12.0000    8.0000
    1.0000    9.0000   27.0000   27.0000
    1.0000   12.0000   48.0000   64.0000
```

   (a) First of all, what function did we just interpolate?

   (b) If we interpolate a cubic polynomial with a spline, what do we get?

   (c) The way Matlab delivers the coefficients was explained in Lab # 5. In short and with an example, the third line corresponds to the coefficients in the third piece (in this case in the interval $[2,3]$ given in the following order

$$(x-2)^3 + 6(x-2)^2 + 12(x-2) + 8.$$

   Check by hand that the polynomial we get in each piece is always the same, namely x^3.