MATH 353: Engineering Mathematics III – Section 012

Spring 2013 (F.–J. Sayas) Homework #6 Due April 5

1. (Review) Write the Lagrange formula for the interpolation polynomial at the points

$$(0,1), \qquad (1,2), \qquad (2,\tfrac{1}{7}), \qquad (3,-\tfrac{2}{3}).$$

2. (By hand) Find the degree of precision of the formula

$$\int_{-1}^{1} f(x)\mathrm{d}x \approx f(-1/\sqrt{3}) + f(1/\sqrt{3}).$$

(**Hint.** Compare the exact and approximate values for $f(x)=1$, $f(x)=x$, $f(x)=x^2$, ... until they are different.)

3. (By hand) Compute α,β,γ so that the following approximation

$$\frac{\alpha f(x_0-h)+\beta f(x_0)+\gamma f(x_0+2h)}{h} \approx f'(x_0)$$

is of order two. (**Hint.** Subtitute with the Taylor expansions

$$\begin{aligned} f(x_0+2h) &= f(x_0)+2hf'(x_0)+\frac{(2h)^2}{2}f''(x_0)+\frac{(2h)^3}{6}f'''(c_1),\\ f(x_0-h) &= f(x_0)-hf'(x_0)+\frac{h^2}{2}f''(x_0)-\frac{h^3}{6}f'''(c_2)\end{aligned}$$

and try to find conditions satisfied by α,β and γ.)

4. (Computer – to figure out) Find out how to add labels to the x and y axis in a figure. Show a log-log plot of the values (h,h^2) for $h=1/2^n$ and $n=1,\ldots,10$, where the x axis is tagged as *meshsize* and the y axis is tagged as *errors*

5. (Computer) In the last Lab you should have finished with three functions (the first one was provided)

```
midpointrule(f,interval,m)
traepozidrule(f,interval,m)
simpsonrule(f,interval,m)
```

Give the code for the last two ones. *The code has to contain the help lines!*

6. (Computer) In the same graph you are going to show the errors of the three formulas when you try to compute the integral

$$\int_0^1 xe^x\mathrm{d}x,$$

using $m2,4,8,16,32,....,128$ subdivisions of the interval. Plot the errors in the same loglog plot using circular markers and lines: the error for the midpoint rule in blue, trapezoid in red, Simpson in black.

7. (Computer) With your code `simpsonrule` and $m = 1$, check that the Simpson rule integrates exactly

$$\int_0^3 (x^3 - 4x + 1)\mathrm{d}x.$$

8. (Computer) If $M_h(f)$ is the composite midpoint rule, $T_h(f)$ is the composite trapezoid rule and $S_h(f)$ is the composite Simpson rule, choose a function (not a polynomial) and m and check that

$$S_h(f) = \frac{2}{3}M_h(f) + \frac{1}{3}T_h(f),$$

that is, compare the result of the computations in both sides of the equality above.

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Spring 2013 (F.–J. Sayas) Homework #5 Due March 22

1. (By hand) Exercise 5.1 in the book (page 252)
2. (By hand) Exercise 5.2 in the book (page 252)
3. (Review - do by hand) Assume that we are working with floating point numbers with only 5 digits. Every single operation you perform has to be rounded-off to five significant digits (the exponents are not an issue). Show what you get when you compute

$$\frac{f(1.0001)-f(1)}{0.0001} \qquad \frac{f(1.00001)-f(1)}{0.00001}$$

   if $f(x)=x^3$.

4. (Computer) We want to check this double forward difference formula to approximate the derivative.

$$\frac{-f(x_0+2h)+4f(x_0+h)-3f(x_0)}{2h}\approx f'(x_0)$$

   (a) Write a testing device (like the functions `testFwdDiff` and `testCentDiff` in the Lab) to test that

$$E_h=\left|\frac{-f(x_0+2h)+4f(x_0+h)-3f(x_0)}{2h}-f'(x_0)\right|\approx C\,h^2$$

   (b) Check it with the function $f(x)=\exp(x)$ at $x_0=1$. Do experiments with $h=0.1, 0.01, 0.001$ and verify that you get what you expect.

   (c) Do a loglog plot of (h, E_h) where $h=[0.5, \quad 0.25, \quad 0.125, \quad ..., \quad 2^{-10}]$. In the loglog plot, use circular markers and lines. On top of this plot, show the line (h,h^2) to verify that the predicted behavior.

5. (By hand) Recall the Taylor expansion formula

$$f(x+h)=f(x)+hf'(x)+h^2\tfrac{1}{2}f''(x)+h^3\tfrac{1}{6}f'''(x)+h^4\tfrac{1}{24}f^{iv}(c),$$

   where c is unknown between x and $x+h$. Use it to show that

$$\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}=f''(x_0)+\mathcal{O}(h^2),$$

   where by $\mathcal{O}(h^2)$ we mean something that can be bounded above by Ch^2.

6. (Computer) Follow the steps of Problem 4 to show that

$$E_h=\left|\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}-f''(x_0)\right|\approx C\,h^2$$

   for the function $f(x)=\log(x)$ at $x_0=1$ (Recall that for us –and for Matlab– log is the natural logarithm.)