MATH 353: Engineering Mathematics III – Section 012

Spring 2013 (F.–J. Sayas) Homework #8 Due April 29

> **Important.** Whenever you write a function, don't forget to include the help lines. The axes in all plots have to be labeled. Absolutely no late homework.

1. (Review – by hand) The following errors correspond to three numerical integration methods applied to compute a given integral with $n = 2, 4, 8, 16, 32, 64$. The errors are organized by column:

0.031671705324960	1.952492442012559	2.000000000000000
0.002154087736268	0.758645757892458	0.523753778993720
0.000137626485773	0.316070565546656	0.132554010550631
0.000008649619087	0.141607886822363	0.033241722501987
0.000000541355236	0.066657646887053	0.008316917839812
0.000000033846503	0.032289766968436	0.002079635476381

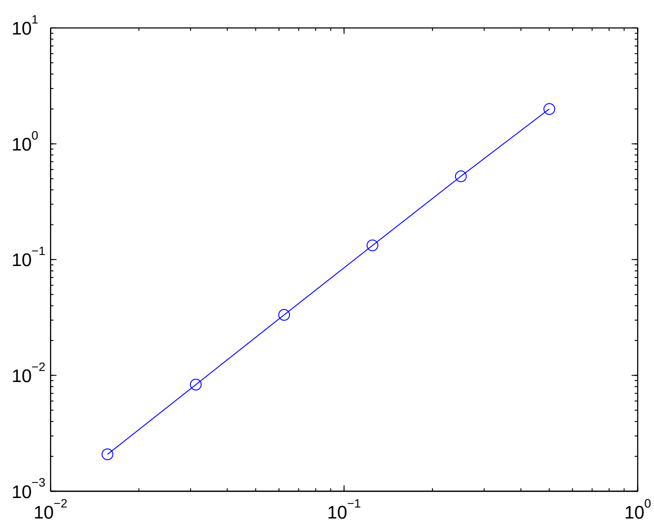
The error of the methods is assumed to behave like $E_h = \mathcal{O}(h^p)$ for some p. What are the orders of convergence p and why?

2. (Review – by hand) Consider the system of differential equations

$$\begin{cases} u' = -2u + v + t, & 0 \le t, \\ v' = u - 2v - t^2, & 0 \le t, \\ u(0) = 1, \\ v(0) = 0. \end{cases}$$

Give two steps of Euler's method with time step $h = 0.1$.

3. (By hand) Here is a loglog plot of some errors E_h corresponding to $h = 1/2, 1/4, 1/8, 1/16, 1/32, 1/64$. Find what point corresponds to the experiment with $h = 1/4$. Also, what is the slope if the line in the graph? What does that slope sat about how E_h behaves?



4. (Computer) Define the matrix and the vector

$$A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 4 & 1 & 1 \\ 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 4 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 7 \\ -3 \\ 3 \\ -3 \end{bmatrix}.$$

Solve it using the Matlab backslash command (see Lab # 9).

5. (Computer) Using the command `diag`, write the instructions needed to construct the $N \times N$ tridiagonal matrix

$$\begin{bmatrix} 4 & 1 & & & \\ -1 & 4 & 1 & & \\ & -1 & 4 & 1 & \\ & & \ddots & \ddots & \ddots \\ & & & -1 & 4 \end{bmatrix}$$

for any given N. (**Hint.** The command `ones` is also useful for this.)

6. (Computer) Using the Matlab command `diag` and the command `ones`, write the instructions that are needed to construct the $N \times N$ matrix

$$\begin{bmatrix} 5 & -1 & -1 & \dots & -1 \\ -1 & 5 & -1 & \dots & -1 \\ -1 & -1 & 5 & -\dots & -1 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -1 & \dots & -1 & -1 & 5 \end{bmatrix}$$

7. (Computer) Let $f(x) = \cos(2x)$. Write the Matlab commands needed to produce the *column* vector

$$\mathbf{f} = h^2 \begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{bmatrix} \qquad x_i = h\,i, \qquad i = 1, \dots, N, \qquad h = \frac{2\pi}{N+1},$$

for any given N.

8. (Computer) **A rotating planet.** Given a massive sun and a much smaller object subject to its graviational force, the equations of motion for the smaller object are (with dimensionless variables):

$$x'' = \frac{x}{(x^2 + y^2)^{3/2}}, \qquad y'' = \frac{y}{(x^2 + y^2)^{3/2}}.$$

The sun is located at $(0, 0)$ and assumed not to move. The motion is determined by initial conditions

$$x(0) = 1, \quad y(0) = 0, \quad x'(0) = 0, \quad y'(0) = 1.$$

- Write the previous system as a system of four first order differential equations in the variable

$$\mathbf{z} = (x, y, v_x, v_y) \qquad v_x = x', \qquad v_y = y'.$$

- Write a script where you run the simulation of the motion of the planet. Plot the orbit of the plane (it is given by the first two unknowns of the system).