
MATH 353: Engineering Mathematics III – Section 012

Spring 2013 (F.-J. Sayas)

Lab # 2

February 15

Your first task today is going to be an easy one, but you should get this done as soon as possible:

- Open Matlab and move to the Desktop or to a folder where you can find your work at the end of the day.
- Set up the diary
- Click on Edit and copy the following piece of code. Be very careful of typing all symbols correctly. Matlab will help you with indentation. Don't fight it back! (Don't copy-paste from the pdf file. I want you to type this up!)
- Once you are done, save it. The file should be called `bisection.m`. You do not need to add the `.m` extension. MATLAB will do that for you.

```
function root=bisection(f,a,b,tol)

% root=bisection(f,a,b,tol)
% Input:
%   f      : function
%   [a,b]  : limits of interval
%   tol    : tolerance
% Output:
%   root   : approximated root in (a,b) by bisection method

fa=f(a); fb=f(b);
if fa*fb>=0
    error('The bisection method cannot find a root in this interval')
end
while (b-a)/2>tol
    c=(a+b)/2; fc=f(c);
    if fc==0
        break
    elseif fa*fc<0
        b=c; fb=fc;
    else
        a=c; fa=fc;
    end
end
root=(a+b)/2;
return
```

The MATLAB you should know to understand this.

- What an m-file is and how the word `function` defines a function stored in a MATLAB file (an m-file)
- The fact that the first commented lines are the help. Go to the MATLAB window and type `help bisection` to see what happens

- The meaning of the decision structures

```

if (conditions)          if (conditions)          if (conditions)
    ...
end                      else
                          ...
                          elseif (conditions)
                          ...
                          else
                          ...
                          end
end                      end

```

Note that we write `elseif` and not `else if`. The latter means something different.

- How to use a `while` loop
- How to use `break` to leave a `while` loop
- The comparison symbol `==`

Exercises

1. Type

```
help bisection
```

What happens?

2. Find all three roots of the polynomial

$$p(x) = x^3 + 2x^2 - 1.$$

Give them back with ten digits. (In order to locate where approximately the roots are, you can make a plot and look for intervals to start the bisection method at.)

3. Download the code `fixedpoint.m` from my website. We want to find the fixed point

$$x + \cos x - \sin x = x.$$

(This fixed point is $r = \pi/4$.) Run the following lines. Can you understand what is happening in each of them? Note that in the last one we are not providing variable for the two pieces of output. Matlab then returns only the first one.

```

>> f = @(x) x+cos(x)-sin(x);
>> [fp, xp]=fixedpoint(f,0,10,1e-3)
>> [fp, xp]=fixedpoint(f,0,10,1e-4)    % not enough iterations?
>> r=fixedpoint(f,0,20,1e-6)

```

4. Continuing with the previous example, since we know the fixed point, we are going to look at errors and see if they behave as the theory predicts. Here's what the theory predicts:

If r is a continuously differentiable function and $|g'(r)| = s < 1$, then from sufficiently close to r the fixed point iteration converges and

$$\lim_{i \rightarrow \infty} \frac{|x_{i+1} - r|}{|x_i - r|} = |g'(r)|.$$

In our case, $|g'(r)| = |1 - \sqrt{2}|$. Let's see if you can follow what these lines mean:

```
>> [fp,x]=fixedpoint(f,0.2,30,1e-10);
>> err=abs(x-pi/4)      % What is this?

      % Results not displayed

>> err(2:end)./err(1:end-1)    % And this?

      % Result not displayed.
      % Is this sequence converging to the expected value?
```

5. If you have understood the previous process you will be able to repeat it for the function

$$g(x) = \frac{1}{x} + \frac{4}{5}x.$$

- Check (numerically) that $\sqrt{5}$ is a fixed point.
- Start the iteration at $x_0 = 2$ and check that it converges.
- What is the rate of convergence? Is it what the theory predicts? (The rate of convergence is $\lim_{i \rightarrow \infty} e_{i+1}/e_i$, where $e_i = |x_i - r|$.)

6. **Faster and faster.** Repeat the previous exercise for

$$g(x) = \frac{x}{2} + \frac{5}{2x}.$$

The fixed point is the same, but the iteration is much faster. Why?