MATH 353: Engineering Mathematics III – Section 012

Spring 2013 (F.–J. Sayas) Lab # 3 February 22

Open Matlab and move to the Desktop or to a folder where you can find your work at the end of the day. Set up the `diary`. Type these two lines

```
>> format long
>> format compact
```

Download the functions `fixedpoint.m`, `newton.m` and `secant.m` from my website.

1. Before we start trying things out, type these two lines exactly as they are.

```
>> list=1:0.5:5;
>> trend=list(2:end)./list(1:end-1);
```

What is in `trend`? You should be able to figure it out before you have a look at it.

2. Run the following lines now:

```
>> p = @(x) (x+5)./(x+1);
>> [fix,hist]=fixedpoint(p,1.5,10,1e-4)
>> exact=sqrt(5);    % exact solution
>> err=abs(hist-exact)
>> err(2:end)./err(1:end-1)
```

Since we know that `fixedpoint` is using a fixed point iteration. What can we know about convergence of the iteration? What is the rate of convergence? What is $|p'(r)|$?

3. When you are going to repeat the same kind of experiment several times, the following strategy is useful:

- Open a blank file to write on (Go to Edit and New or click on the blank page icon)
- Copy the following lines (comments are optional but highly recommended).

```
% This is not a function. This a script
% There's no input and no output.

p = @(x) 8./(x+2);    % looking to solve r=p(r)
exact = 2;                    % exact fixed point

[fix,hist]=fixedpoint(p,0.1,40,1e-10);  % fixed point iteration
err=abs(hist-exact)                      % errors
err(2:end)./err(1:end-1)                 % trend of the errors

% It's a good idea to add comments to remember what you did
```

- Save the file as `scriptFebruary22`
- Go to the main window and run this

  ```
  >> scriptFebruary22
  ```

  What did we just do? What was the rate of convergence?

The main difference between a *script* and a *function* is that functions have input and output, while scripts are like lines you run on the main window.

4. **Newton's method and quadratic convergence.** We want to compute zeros of

$$f(x) = \cos(x^2) - \sin(x^2)$$

using Newton's method. We then need

$$f'(x) = -2x(\sin(x^2) + \cos(x^2)).$$

In a new script:

- Define the functions `f` and `fp` containing f and f'.
- Call Newton's method, starting at $x_0 = 0.4$, taking at most 5 iterations and tolerance 10^{-10}.
- We are trying to approximate the value $r = \sqrt{\pi/4}$. Compute the values $e_i = |x_i - r|$.
- Compute the sequence

$$\frac{e_{i+1}}{e_i}$$

  What does it converge to? (If you get this right, you'll have seen that this the sequence x_i converges faster than linearly.)
- Compute the sequence

$$\frac{e_{i+1}}{e_i^2}.$$

  How does this show that Newton's method converges quadratically?

5. Run the `secant` method code with starting values $x_0 = 0$ and $x_1 = 1$ and check that it gets to the right solution.

6. Open the file `secant` and make a modification so that it outputs the history of iterations as well. At the end you should get a function that works like this:

```
[r,hist]=secant(f,x0,x1,tol,itmax)
```

7. Once you have the modification for the secant code, use it to see how the error behaves in the previous example

$$f(x) = \cos(x^2) - \sin(x^2), \quad r = \sqrt{\frac{\pi}{4}}, \quad x_0 = 0, \quad x_1 = 1, \quad \texttt{tol} = 10^{-10}, \quad \texttt{itmax} = 6.$$

In particular I want you to check that

$$\frac{e_{i+1}}{e_i} \to 0 \qquad \frac{e_{i+1}}{e_i^2} \to \infty.$$

(In the second case, you will not get to see the quotient grow all the way to ∞, but it'll grow.) *Write all the instructions in a script.*

Matlab has its own function to compute roots. It is called `fzero` (as in function-zero). It uses Brent's method, which is a mix of several methods. If you want your roots, let Matlab do the job.