MATH 353: Engineering Mathematics III – Section 012

Spring 2013 (F.–J. Sayas) Lab # 8 April 12

Open Matlab and *move to the Desktop or to a folder where you can find your work* at the end of the session. Type these lines

```
>> diary myworkApril12
>> format long
>> format compact
```

Download the function `rk4.m` and the script `scriptApril8and10.m` from my website.

1. First of all, let us check that the RK4 method is actually of order four. Take the equation

$$\frac{y'}{2y^2}+t+1=0, \qquad 0\le t\le 2, \qquad y(0)=1,$$

whose exact solution is $y(t)=\frac{1}{(t+1)^2}$. Using $n=10,20,40,80,...,2560$ points, compute, tabulate and plot the errors

$$E_h=\max_{0\le i\le n}|w_i-y(t_i)|$$

and check that $E_h=\mathcal{O}(h^4)$. **Warning.** The function `rk4` returns a column vector in `w`, so be careful when you compare with `y(t)`, `t` being a row vector. The solution is very simple: `t'` is a column vector with the same values as `t`.

2. Using the example in the script `scriptApril8and10`, carry out the necessary modifications to code the damped forced pendulum equation

$$\theta''+\frac{g}{L}\sin\theta+d\,\theta'=m(t) \qquad 0\le t\le T_{\max}, \qquad \theta(0)=\theta_0, \qquad \theta'(0)=0.$$

The way you have to do this is:

- Set up the physical parameters: g, L, d, the final time $T_{\max}$, and an amplitude parameter A. (See values below.)
- Define the forcing term $m(t)$ as a function of the variable t. Take

$$m(t)=A\sin t.$$

- Define then the function $f(t,y)$ for the associated first order system.
- Run RK4 and plot the results as shown in the script. Do you understand all the plotting commands? For this first experiment take:

$$g=9.81, \qquad L=0.3, \qquad d=0, \qquad A=0, \qquad T_{\max}=5.$$

  You should see periodic motion.
- Repeat again with $d=1$. Can you see the effect of the damping parameter?

- Repeat again with $d = 0$ and $A = 5$.

3. **A rotating planet.** Given a massive sun and a much smaller object subject to its graviational force, the equations of motion for the smaller object are (with dimensionless variables):

$$x'' = \frac{x}{(x^2 + y^2)^{3/2}}, \qquad y'' = \frac{y}{(x^2 + y^2)^{3/2}}.$$

The sun is located at $(0, 0)$ and assumed not to move. The motion is determined by initial conditions

$$x(0) = x_0, \quad y(0) = y_0, \quad x'(0) = v_0^x, \quad y'(0) = v_0^y.$$

- Write the previous system as a system of four first order differential equations in the variable

$$\mathbf{z} = (x, y, v^x, v^y) \qquad v^x = x', \qquad v^y = y'.$$

Write a script where you run the simulation of the motion of the planet and plot it using `comet`. Run several simulations to see if by changing the initial conditions you can find different kinds of orbits.