MATH 508 – Spring 2012

What to review for the in-class part of the midterm exam

Manipulation of complex numbers

- Sum, subtraction, multiplication and division of complex numbers
- Real and imaginary part of a complex number; representation of complex numbers in the complex plane
- Conjugate, absolute value, argument and principal argument of a complex number. How to compute them, what they mean geometrically, and their basic properties (including the triangle inequality)
- Polar form of a complex number
- Understanding how the polar form of a complex number is affected by conjugation and inversion
- Geometric interpretation of adding complex numbers, multiplying by a complex number, squaring a complex number, taking a power of a complex number
- Understanding simple sets in the complex plane (horizontal and vertical lines and strips, disks, circumferences, quadrants and other angular sectors)

Important functions

- Exponential of a complex number. Its main properties: it is periodic in the imaginary direction, it never vanishes,…
- Understanding the geometric effect of the exponential
- How to compute the n-th roots of a complex number. Geometric localization of all n-th roots of a complex number
- The sine, the cosine, the hyperbolic sine and the hyperbolic cosine.
- Polynomials: how to factorize them; how to write them in Taylor form
- Rational functions: zeros and poles; cancellation of common zeros and poles; decomposition in partial fractions (what it means and how it is computed)
- How to compute all logarithms of a complex number and the principal logarithm (Log)

Analyticity

- Limits of complex functions
- Understanding where and why functions like Arg or Log are discontinuous
- What is a limit at infinity and what it means that a limit takes the infinity value (it's just a question of the absolute value)
- Complex differentiable functions: what the concept means and the basic rules of differentiation, including the Leibnitz rule (differentiation of a product) and the chain rule
- Derivatives of elementary functions: polynomials, rational functions, the exponential and its related functions (sin, cos, sinh, cosh,…), the logarithm
- Analytic functions at a point (functions that are differentiable at all points in a neighborhood of the point) and at a set
- Entire functions (functions that are analytic in the complete complex plane)
- How to decompose a complex function in its real and imaginary parts (written as functions of two variables each)
- The Cauchy-Riemann equations. How to use them to show that a given function is analytic.
- Examples of how to use the CR equations to prove easy properties of analytic functions: if a function has constant real part it needs to be constant, etc.

PARTICULAR THINGS TO REVIEW

- The two quizzes
- The assigned homework problems
- All examples/exercises that I have worked out in class
- The recommended homework problems

WHAT YOU ARE ALLOWED TO BRING TO THE EXAM

- The textbook. (No other books are allowed.)
- Your own handwritten notes.
- Don't overdo it with the amount of material you bring to the exam. Too much material and/or material you are not familiarized with (copies of a classmate's notes, no matter how good they are) is just going to distract you. Think that you will not find in the notes what's not already in your head. At least, not during the exam.