MATH 508 – Spring 2012

What to review for the in-class part of the final exam

Manipulation of complex numbers

- Sum, subtraction, multiplication and division of complex numbers
- Real and imaginary part of a complex number; representation of complex numbers in the complex plane
- Conjugate, absolute value, argument and principal argument of a complex number. How to compute them, what they mean geometrically, and their basic properties (including the triangle inequality)
- Polar form of a complex number
- Understanding how the polar form of a complex number is affected by conjugation and inversion
- Geometric interpretation of adding complex numbers, multiplying by a complex number, squaring a complex number, taking a power of a complex number
- Understanding simple sets in the complex plane (horizontal and vertical lines and strips, disks, circumferences, quadrants and other angular sectors)

Important functions

- Exponential of a complex number. Its main properties: it is periodic in the imaginary direction, it never vanishes,…
- Understanding the geometric effect of the exponential
- How to compute the n-th roots of a complex number. Geometric localization of all n-th roots of a complex number
- The sine, the cosine, the hyperbolic sine and the hyperbolic cosine.
- Polynomials: how to factorize them; how to write them in Taylor form
- Rational functions: zeros and poles; cancellation of common zeros and poles; decomposition in partial fractions (what it means and how it is computed)
- How to compute all logarithms of a complex number and the principal logarithm (Log)

Analyticity

- Limits of complex functions
- Understanding where and why functions like Arg or Log are discontinuous
- What is a limit at infinity and what it means that a limit takes the infinity value (it's just a question of the absolute value)
- Complex differentiable functions: what the concept means and the basic rules of differentiation, including the Leibnitz rule (differentiation of a product) and the chain rule
- Derivatives of elementary functions: polynomials, rational functions, the exponential and its related functions (sin, cos, sinh, cosh,…), the logarithm
- Analytic functions at a point (functions that are differentiable at all points in a neighborhood of the point) and at a set
- Entire functions (functions that are analytic in the complete complex plane)
- How to decompose a complex function in its real and imaginary parts (written as functions of two variables each)
- The Cauchy-Riemann equations. How to use them to show that a given function is analytic.
- Examples of how to use the CR equations to prove easy properties of analytic functions: if a function has constant real part it needs to be constant, etc.

Integration

- How to parametrize the simplest curves (line segments,portions of circles) and how to reverse a parametrization
- Definition of integral using a parametrization
- Simple properties: linearity, change of sign due to orientation
- Path independence of integrals of functions that have an antiderivative. How to use it to compute an integral
- Simply connected regions. Integrals of analytic functions on simply connected regions vanish
- Important integral formulas and arguments:
    - Cauchy's Integral Formula (integral of an analytic function divided by $z-z0$)
    - Its generalization to higher powers (leading to evaluation of higher order derivatives)
    - How to move from an integral over one loop to separated integrals over smaller loops surrounding separate singularities

Series

- Sum of the simplest series (geometric series, exponential,…)
- The ratio test for series (how it detects converging but also diverging series)
- Taylor series expansion of analytic functions around a point. It converges in a disk as large as you can fit in the domain of analyticity of the function
- Power series: concept of radius of convergence (convergence inside, divergence outside)
- How power series become Taylor series of the function they define and where to find information about the function in the coefficients
- Factorization of zeros of order m of analytic functions
- Laurent series for functions that are analytic in an annular domain
- How to get different Laurent series representations of the same function in different concentric annuli. How the convergence regions are determined by singularities
- Classification of singularities (avoidable, poles and essential singularities) by looking at the Laurent series expansion in a punctured disk around the singularity
- Factorization of poles in the denominator

Residues

- Definition of residue: the coefficient with the first negative power in a Laurent series expansion for a function that analytic around a point but not in the point
- Formula to compute the residue of a function at a pole
- Cauchy's residue theorem

PARTICULAR THINGS TO REVIEW

- The five quizzes
- The assigned homework problems
- All examples/exercises that I have worked out in class
- The recommended homework problems

WHAT YOU ARE ALLOWED TO BRING TO THE EXAM

- The textbook. (No other books are allowed.)
- Your own handwritten notes.
- Don't overdo it with the amount of material you bring to the exam. Too much material and/or material you are not familiarized with (copies of a classmate's notes, no matter how good they are) is just going to distract you. Think that you will not find in the notes what's not already in your head. At least, not during the exam.