Simple properties of subsets

by F.J.Sayas, for MATH 600

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> As usual, will use the symbol $\subset$ understanding that when $U \subset V$, we admit the possibility of $U = V$.

In this short worksheet we will be working on subsets of a set X. Given $A \subset X$ we write

$$A^c = \{x \in X \,:\, x \notin A\}$$

to denote the complement of A.

(a) Show that $(A^c)^c = A$.

(b) Show that $A \subset B$ if and only if $B^c \subset A^c$.

(c) Let I be any set of indices and let $A_\alpha \subset X$, $\alpha \in I$. Show that

$$\Big(\bigcup_{\alpha\in I} A_\alpha\Big)^c = \bigcap_{\alpha\in I} A_\alpha^c$$

and

$$\Big(\bigcap_{\alpha\in I} A_\alpha\Big)^c = \bigcup_{\alpha\in I} A_\alpha^c.$$

These two equalities are know as DeMorgan's Laws.

(d) Let I be any set of indices and let $A_\alpha \subset X$, $\alpha \in I$ and $B \subset X$. Show that

$$\Big(\bigcup_{\alpha\in I} A_\alpha\Big) \cap B = \bigcup_{\alpha\in I}(A_\alpha \cap B)$$

and

$$\Big(\bigcap_{\alpha\in I} A_\alpha\Big) \cup B = \bigcap_{\alpha\in I}(A_\alpha \cup B).$$

These two equalities are called the distributive laws.