MATH 612: CM4ES&FM

Spring'14 | Coding assignment # 1 | Due March 7

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1. Exploring the numerical rank. Consider two collections of points

$$\mathbf{x}_i \in \mathbb{R}^3, \qquad i = 1, \ldots, 20, \qquad \mathbf{y}_j \in \mathbb{R}^3, \qquad j = 1, \ldots, 30.$$

   The points $\mathbf{x}_i$ are taken randomly in the cube $[-1/2, 1/2]^3$. The points $\mathbf{y}_j$ are taken randomly in the cube $(d, 0, 0) + [-1/2, 1/2]^3$, with $d > 1$. Construct the matrix

$$a_{ij} = \frac{e^{\imath\, k\, d_{ij}}}{d_{ij}} \qquad d_{ij} := \|\mathbf{x}_i - \mathbf{y}_j\|_2.$$

   Here k is a positive number and $\imath$ is the imaginaty unit. Using a rank reduction algorithm (fixing the rate), estimate how the effective rank of A behaves as a function of d and k. In the case of k, we are interested in k close to zero and growing towards infinity.

   As a challenge try to construct the matrix without looping over n and m. (Learn how to use `bsxfun` to do this.)

2. Code Householder's method to obtain a QR decomposition of any matrix with maximum rank by columns. The input should be the matrix A. The requested format is

```
[U,R]=householder(A)
```

   where R is upper triangular with the same shape as A, and U is an $m \times n$ matrix with the collection of unit vectors $u_1, \ldots, u_n$ as columns. The unitary matrix in the QR decomposition is

$$Q = H_{u_1} \ldots H_{u_n}.$$

   (The vectors in Q appear in the reverse order as they were produced.) The code can use the function `householdertransf` created in class. To compute Q for testing, you can use `householdertransf` applied to an identity matrix. Note however, that the progressive Householder reflections are known in advance not to modify some of the columns at the beginning of the matrix. This should be taken into account.

> I'll give some coding tips on Monday. It'd be great if you had already thought about this by then.