MATH 612
Computational methods for equation solving and function minimization
Exam # 1 – Fast Rounds

Spring 2014 – University of Delaware

Right now

- Write your name in the first page
- Write a 3 digit number in the box provided
- Write the same 3 digit number in the box in the second page
- Ready, set,...

Problem 1

What is the result of the following commands?

```
list=1:0.2:2;
list=list(end:-1:1)
```

Problem 2

What is the result of the following commands?

```
A=[1 2 3 4;5 6 7 8;9 10 11 12];
A=A(:,end:-1:1)
```

Problem 3

What is the result of the following commands?

```
A=[1 2 3 4;5 6 7 8]';
A(3,:)=[]
```

Problem 4

What is the result of the following commands?

```
x=[1 2 3]';
sqrt(x'*x)-sqrt(sum(abs(x).^2))
```

Problem 5

If `x` stores a row or column vector and we compute,...

```
sum(abs(x))
max(abs(x))
```

what have we computed?

Problem 6 (counts double)

We have the full and reduced SVD of an $m \times n$ matrix

$$A = U\Sigma V^* = \widehat{U}\widehat{\Sigma}\widehat{V}^*.$$

The columns of U are the vectors $u_1, \ldots, u_m$. The columns of V are $v_1, \ldots, v_n$. The matrix $\widehat{\Sigma}$ is $r \times r$. Give the results of the following computations:

- $u_i^* u_j$
- $V^* V$
- Av_j for $j \leq r$
- Av_j for $j \geq r + 1$.

Problem 7

When we say that a matrix Q is unitary, what do we mean?

Problem 8

For this given matrix

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix},$$

what is Σ in the SVD

$$A = U \Sigma V^* \quad ?$$

Problem 9

Define the Frobenius norm of a matrix

Problem 10

We define

$$\|A\|_p = \sup_{0 \neq x \in \mathbb{C}^n} \frac{\|Ax\|_p}{\|x\|_p}.$$

Show that if $\lambda \in \sigma(A)$, then $|\lambda| \leq \|A\|_p$.

Problem 11

The matrix A^*A has a unique dominant eigenvalue, which is real. Why?

Problem 12

Show that

$$x \in \text{null}(A^* A) \qquad \Longleftrightarrow \qquad x \in \text{null}(A).$$

Problem 13

The reduced QR decomposition of a matrix A is

$$A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$$

Compute a full QR decomposition of A.

Problem 14

We have the reduced QR decomposition of a matrix A with full rank by columns:

$$A = \widehat{Q}\widehat{R}.$$

Show what you need to do to solve the least squares problem

$$\text{minimize } \|b - Ax\|_2.$$

Problem 15

Define the condition number of a square invertible number with respect to the p norm.

Problem 16

Let Q be a unitary matrix. What is its condition number in 2-norm? Why?

Problem 17

Let $u \in \mathbb{C}^m$ be a non-zero column vector and let

$$H = I - 2u\,u^*.$$

Show that $H^{-1} = H$ if and only if $\|u\|_2 = 1$.

Problem 18

What is the result of the following computation?

```
[Q,R]=qr(randn(5,5));
Q'*Q
```

Problem 19

What method is this and what does it compute?

```
for j=1:n
    v=A(:,j);
    for i=1:j-1
        R(i,j)=dot(Q(:,i),v);
        v=v-R(i,j)*Q(:,i);
    end
    R(j,j)=norm(v);
    Q(:,j)=(1/R(j,j))*v;
end
```

Problem 20

If A is $m \times n$ and $k \leq \min\{n, m\}$, what size is B? What is the rank of B?

```
[U,S,V]=svd(A);
UU=U(:,1:k);
VV=V(:,1:k);
SS=S(1:k,1:k);
B=UU*SS*VV';
```

Problem 21

What do we understand by the normal equations associated to a system $Ax = b$?

Problem 22

What is the result of the following computation?

```
a=[2 4 6]./[2 2 3]*0.5;
a'*a
```

Problem 23

What is the result of the following computation?

```
D=[3 0 0;0 -1 0;0 0 4];
norm(D,inf)
```

Problem 24

What is the result of the following computation?

```
A=[1 2 3 4;5 6 7 8;9 10 11 12];
A([1 3],:)=A([3 1],:)
```

EXTRA QUESTIONS

Problem 25

What do we mean when we say that P is a projector?

Problem 26

What are the (possible) eigenvalues of a projector?

Problem 27

Let A be a matrix with full rank by columns. How is the matrix

$$P = A(A^*A)^{-1}A^*$$

related to a projector? (Do not prove anything! Just state as many facts as you can!)

Problem 28

What are `k1` and `k2`?

```
b=[0 1 2 0 -3]';
k1=find(b~=0,1)
[~,k2]=max(abs(b))
```

Problem 29

What is the result of these lines of code?

```
A=[1 2;3 4];b=[5 6]';
A=[A b]
```