MATH 612: CM4ES&FM

Spring'14 | Homework # 4 | Due April 28

1. Problem 33.1
2. Problem 33.2
3. Problem 38.3
4. Problem 38.5
5. Let $\varphi : \mathbb{R} \to \mathbb{R}$ be differentiable.

   (a) Show that φ is convex if and only if

   $$\frac{\varphi(t_1) - \varphi(t_0)}{t_1 - t_0} \le \frac{\varphi(t_2) - \varphi(t_0)}{t_2 - t_0} \qquad \forall t_0, t_1, t_2, \qquad t_0 < t_1 < t_2.$$

   (Hint. Rename x_0, x_1 and τ in the definition of convexity and do some algebra.)

   (b) Show that the convexity of φ is also equivalent to

   $$\frac{\varphi(t_1) - \varphi(t_0)}{t_1 - t_0} \le \frac{\varphi(t_2) - \varphi(t_1)}{t_2 - t_1} \qquad \forall t_0, t_1, t_2, \qquad t_0 < t_1 < t_2.$$

   (Hint. If $0 < \beta < \gamma$, then $b/\beta \le c/\gamma$ if and only if $b/\beta \le (c - b)/(\gamma - \beta)$.)

   (c) Finally, show that convexity of φ is equivalent to φ' being non-decreasing.

6. Let $f : \mathbb{R}^n \to \mathbb{R}$. Show that

   (a) If f is strictly convex, then f cannot have local maxima.

   (b) If f is convex, then f cannot have strict local maxima.

   (c) There is a convex function f such that f has local maxima.

7. Let $x_d \in \mathbb{R}^m$, $d \in \mathbb{R}^m$, $D \in \mathbb{R}^{m \times n}$, and $C \in \mathbb{R}^{m \times m}$ be invertible. Consider the function $f : \mathbb{R}^n \to \mathbb{R}$ given by

   $$f(u) = \tfrac{1}{2}|x - x_d|^2 + \tfrac{1}{2}|u|^2, \qquad \text{where } Cx = Du + d.$$

   Show that f is strictly convex and compute its gradient. If possible, write a simple formula for $\nabla f(u)$ in terms of u and x.

8. Code and test GMRES