MATH 612
Computational methods for equation solving and function minimization
Exam # 2 – Fast Rounds

Spring 2014 – University of Delaware

Right now

- Write your name in the first page
- Write a 3 digit number in the box provided
- Write the same 3 digit number in the box in the second page
- Ready, set,...

Problem 1

What is the result of the following commands?

```
>> A=[1 2 3 4;5 6 7 8;9 10 11 12];
>> p=[2 3 1];
>> A=A(p,:)
```

Problem 2

What is the result of the following commands?

```
>> A=[1 2 3;2 3 4;3 4 5];
>> A(2:3,:)=A(2:3,:)-[2;3]*A(1,:)
```

Problem 3

What is the result of the following commands?

```
>> A=[3 4 5;1 2 3];
>> A=[A A(:,3)]
```

Problem 4

We have defined

```
>> A=randn(5);B=randn(5);c=ones(5,1);
```

We want to compute $D = ABc$. Write the MATLAB command that computes this product in a way that reduces the number of floating point operations.

Problem 5

Here's a while loop.

```
c=4;
i=1;
while c>1
    c=c-2;
    i=i+1;
end
```

What's the value of `i` at the end?

Problem 6

What is the result of the following commands?

```
>> A=[1 2 3;4 3 2;3 4 5;6 5 4];
>> A([2 4],:)=A([4 2],:)
```

Problem 7

What is the result of the following commands?

```
>> list=1:2:7;
>> list=list(end:-1:1)
```

Problem 8

What is the result of the following commands?

```
>> f = @(x) x.^2./(1+x);
>> f([1 2 3])
```

Problem 9

Define strictly convex function.

Problem 10

Let f be convex, and let x and y be global minima of f. Show that $(1-\tau)x+\tau y$ is also a minimum for every $\tau \in (0,1)$.

Problem 11

Let f be convex and $\alpha \in \mathbb{R}$. Show that the set

$$\{x \,:\, f(x) \leq \alpha\}$$

is convex.

Problem 12 (counts double)

Let A and B be $s_1 \times n$ and $s_2 \times n$ matrices. Show that the set

$$C = \{x \in \mathbb{R}^n \,:\, Ax \leq b, \quad Bx = c\}$$

is convex. Write the set in the form

$$C = \{x \in \mathbb{R}^n \,:\, Dx \leq f\}$$

for adequate D and f.

Problem 13

Let $f : \mathbb{R}^n \to \mathbb{R}$ be differentiable. Define what we understand by a descent direction at the point x.

Problem 14

In class we have shown that a differentiable function of one variable is convex if and only if

$$f(x) \geq f(y) + f'(y)(x - y) \qquad \forall x, y \in \mathbb{R}.$$

Make a picture that explains what this property means.

Problem 15

Give an example of an strictly convex function that does not attain its minimum. You have to show that the function is strictly convex as part of the solution.

Problem 16

What is the goal of the following iteration? Can you name how the method is called?

for $\nu \geq 1$
    $b = \nabla f(x)$
    $A = Hf(x)$ (Hessian matrix)
    $w = A^{-1}b$
    $\varphi_0 = f(x)$, $\psi_0 = w \cdot b$
    $\tau = \gamma$
    $\varphi_1 = f(x + \tau w)$
    while $\varphi_1 > \varphi_0 + \tau\beta\psi_0$
        $\tau = \tau\gamma$
        $\varphi_1 = f(x + \tau w)$
    end
    $x = x + \tau w$
    stopping criterion
end

Problem 17 (counts double)

We consider the problem of minimizing

$$f(x_1, x_2, x_3) = x_1^2 + 2x_1x_2 + 3x_2^2 + x_3^4$$

subject to

$$x_1 + x_2 + x_3 = 2, \qquad x_1^2 + x_2^2 + x_3^2 = 4.$$

Write the associated Lagrangian and the necessary conditions for minimization.

(Hint. Write the constraints as $f_i(x) = 0$ and include a Lagrange multiplier for each constraint. The conditions are related to finding stationary points for the Lagrangian.)

Problem 18

What exactly do we mean when we talk about a Krylov space? (How many ingredients are involved in the definition of a Krylov space?)

Problem 19

Define Hermitian positive definite matrix

Problem 20

What do we call a Cholesky decomposition of a matrix? For which matrices do these decompositions exist?

Problem 21

We bring a square invertible matrix A and a vector b to an Arnoldi iteration. After n iterations, can you say what have we computed?

Problem 22

The conjugate gradient method solves some systems

$$Ax = b.$$

What are the requirements on A for the CG method to apply?

Problem 23

In the following product A is $m \times m$, Q_n is $m \times n$ with orthonornal columns, and Q_{n+1} is an extension of Q_n with an additional orthonormal column:

$$AQ_n = Q_{n+1} \begin{bmatrix} \alpha_1 & \overline{\beta_1} & & & \\ \beta_1 & \alpha_2 & \overline{\beta_2} & & \\ & \beta_2 & \alpha_3 & \ddots & \\ & & \ddots & \ddots & \overline{\beta_{n-1}} \\ & & & \beta_{n-1} & \alpha_n \\ & & & & \beta_n \end{bmatrix}$$

Looking at the decomposition, what is Aq_j?