MATH806 – Functional Analysis

Catalog description

Fundamental results for normed linear spaces with special emphasis on the theory of Hilbert space, spectral theorems and application to integral and differential equations.

Essential information

- Instructor: Francisco-Javier "Pancho" Sayas
- Lecture times: MW 5:00 – 6:15 in EWG203
- Office hours: By appointment. We will have a two hour weekly problem solving meeting.
- Website: www.math.udel.edu/~fjsayas/math806
- Textbook: Alberto Bressan, *Lecture Notes on Functional Analysis*, AMS Graduate Studies in Mathematics, Volume 143. (We will cover Chapters 2, 4, 5, and 6. The contents of Chapter 1 are expected to be known by the students when taking this class.) Purchasing this book is highly recommended. As a graduate student at UD you have a discount through the AMS website.

Course contents

The following topics will be covered, not necessarily in this order. We will avoid some of the most convoluted proofs and will focus on how to build a large technical toolbox.

- Banach spaces
  - Basic definitions and particular cases (finite dimensions)
  - Linear operators
  - Construction of Frechet spaces
  - Duality and weak convergence
  - The Hahn-Banach theorem
  - Convexity
- Bounded linear operators in Banach spaces
  - The uniform boundedness principle (Banach-Steinhaus)
  - The open mapping theorem
  - The closed graph theorem
  - Adjoint operators
  - Compact operators
- Hilbert spaces
  - Orthogonal projections
  - Orthogonal sequences and Hilbert bases
  - The Riesz-Frechet theorem and applications
  - Weak convergence
- Spectral theory on Hilbert spaces

- o The Fredholm alternative
- o Spectrum of a compact operator
- o The Hilbert-Schmidt theorem
- Banach algebras (time permitting)

Structure of the course

Class and study time will be divided in different activities:

- **Lecturing.** At the time of each lecture the student will be expected to have read a section of the book. The lectures will be self-contained but will only contain sketches of proofs or difficult details that might be hard to understand when reading the book.
- **Quizzing.** Every now and then there will be an unannounced quiz in class. Quizzes will assess on basic concepts (definitions, statements of important theorems) and mathematical techniques (one line proofs)
- **Problem solving.** Homework will not be collected or graded. We will articulate ways of going through the most challenging problems so that you are prepared for tests.
- **Reading.** In addition to take some time to read the sections of the book that we will be covering, I will prepare some documents where you will be challenged to read (and fill the gaps) on theory that we will see later on.

Collaboration and online searches

The material explored in this course is extremely standard. All problems can be easily found solved online. Students are recommended to work their way through the problems, only occasionally trying to find a hint through searches (in this case, it is recommended to have a fast look at the idea and try to reconstruct the solution by working out the details). I strongly recommend working in teams for some of the more challenging problems. In take-home exams all kind of collaboration and online searches will be strictly forbidden.

Evaluation
- 2 in-class exams (dates to be announced): 20% of the grade each
- 2 take-home exams: 20% of the grade each
- Frequent unannounced quizzes, for a total of 20% of the grade.

Attendance policy
Attendance to lectures is mandatory. No make-up quizzes will be given.

Academic honesty

As worded by the University of Delaware, All students must be honest and forthright in their academic studies. To falsify the results of one's research, to steal the words or ideas of another, to cheat on an assignment, or to allow or assist another to commit these acts corrupts the educational process. Students are expected to do their own work and neither give nor receive unauthorized assistance. Any violation of this standard must be reported to the Office of Student Conduct. For more details, check (www.udel.edu/stuguide/15-16/code.html#honesty). Cheating of any kind (even if the student does not take any advantage from it) will be grounds for an F grade.